Comments on "Vibrational Characteristics of Thin-Wall Conical Frustum Shells"

WILLIAM C. L. HU*
Southwest Research Institute, San Antonio, Texas

IN a recent note, Watkins and Clary[1] reported some interesting experimental results on resonant frequencies and nodal patterns of vibrations of truncated conical shells. It was reported that for free-free conical shells, there are a greater number of circumferential nodes (of transverse displacement w) at the major edge than at the minor edge when the shell is in resonant vibration. In Fig. 2 of their note,[1] a sketch of a typical nodal pattern was shown which included two U-shaped nodal lines with both ends at the major edge. The authors remarked that special analytic procedures are needed to account for the difference in the number of circumferential nodes at the two edges.

It is felt that this phenomenon may be explained by the fact that the authors used two electromagnetic shakers to excite the resonant vibrations, with one shaker mechanically attached to each end of a generator (Fig. 2, Ref. 1). Therefore, the reported modes are steady-state forcing vibrations in resonance with some natural frequencies that may, but do not necessarily, simulate the associated normal modes of free vibrations of the tested shell. Since material damping and acoustic damping are always present in vibration experiments, the use of some kind of shaker is essential to keep the vibration continuing for measurement. However, if the purpose of the experiment is to investigate the natural frequencies and associated normal modes of the tested model, the placement of the shaker, or shakers, must be carefully designed so as to effect energy transfer with minimum distortion of the normal mode. One way to check the distortion of the modal shape is by cutting off the energy source of the shakers and examining whether there is any appreciable change of the nodal pattern during the decaying free vibration.

Since the term normal mode denotes nothing but the harmonic component resulting from mathematical decomposition of the homogeneous solution of a linear dynamic system, the nodal pattern of a normal mode of a shell of revolution (with axisymmetric boundary conditions) naturally consists of parallel circles and equispaced meridians, because the axial symmetry of the shell and circumferential periodicity of the vibration motion permit the solution of the problem by separation of variables.[2–5] A recent experimental program on vibrations of conical shells conducted at Southwest Research Institute confirms the foregoing discussion. In these experiments steel shells were excited through harmonically varying magnetic fields located near the opposite ends of a diameter of the major base, and no U-shaped nodal lines are observed in resonant vibrations of free-free or freely supported conical shells.

It might be remarked that there is another important factor in getting an "irregular" nodal pattern, namely, the dynamic system must be such that some distinct normal modes have equal or nearly equal frequencies (which is highly probable in vibrations of plates and shells); then their linear combinations will result in nodal patterns of complex nature. The nodal patterns of a vibrating square plate with free edges furnish a well-known example.

References

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Received December 14, 1964.

* Research Engineer, Department of Mechanical Sciences. Member AIAA.

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[3] Kalnins, A., "Analysis of shells of revolution subjected to symmetrical and nonsymmetrical loads," J. Appl. Mech. **31**, 467–476 (1964).

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Reply by Authors to W. C. L. Hu

J. D. WATKINS* AND R. R. CLARY†
NASA Langley Research Center, Hampton, Va.

THE authors wish to thank Hu for his comments in regard to the conical frustum nodal patterns reported in their recent technical note.

It appears that the issues under discussion are the following: 1) Do the true nodal patterns associated with the normal modes of an idealized isotropic, axisymmetric conical frustum shell always consist of parallel circles and equispaced meridians? 2) If so, are the departures from the idealized case (due to damping, anisotropy, nonlinearities, shaker interferences, boundary restrictions, etc.) inherent in the conical frustum structures tested such that the node lines deviate from the idealized case as indicated in the authors' paper? The authors are not sure of the answer to the first question. Hu and others have suggested that the answer is yes. However, the authors doubt that the idealized mathematical analog is adequate for the general treatment of the physical structures under discussion. If the answer to the first question, a mathematical one, is yes, then the authors believe the answer to the second question, a physical one, is also in the affirmative because they are convinced that the results reported are reliable.

The authors were the first to be surprised at the results of the investigation, and many repetitions of the experiment were necessary to convince them and their associates that the results achieved were valid.

It is the authors' opinion that any influence of shaker position or of the type of shaker used was not sufficient to cause the conical frustum shells to respond in the manner reported. Comparable nodal patterns were obtained when electromagnetic shakers were placed at the following locations: a single shaker at either a major or minor diameter, or at any position along a generator; a shaker attached to each end of either a major or minor diameter; and three shakers spaced evenly along a generator. Effects of mechanical attachment to the shell can be essentially eliminated since similar nodal patterns were obtained when an air shaker[1] was used to excite the shell.

Only for limited cases involving lower-order modes (n = 3,4,6) were the authors able to obtain the mode shapes typical of classical, undamped, linear theory as discussed, for example, by Goldberg.[2] For the higher values of conicity (above 7.4°), classical type modes could be excited only when $n = 3$. In other studies with plates and cylinders, whenever mixed modal response was detected, further application of careful experimental procedures, such as followed in this investigation and in Ref. 3, resulted in exciting the uncoupled modes for the system. However, contrary to the authors' previous experiences, it was not possible, at higher values of

Received January 27, 1964.

* Aerospace Engineer, Vibration and Dynamics Branch, Dynamic Loads Division.

† Aerospace Engineer, Vibration and Dynamics Branch, Dynamics Loads Division. Associate Member AIAA.